Radar Frequency-Comparison Monopulse Validation Using FPGA-Generated Wideband Chirp Waveforms

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Abstract: In this paper, we introduce a frequency-based Direction of Arrival (DoA) estimation technique for wideband radar systems employing Linear Frequency Modulated (LFM) signals. Conventional phase-comparison monopulse is subject to estimation ambiguities, especially in the case of a large baseline distance. In contrast, the proposed frequency-comparison monopulse leverages the time-to-frequency mapping property of LFM signals, enabling unambiguous DoA estimation for any antenna spacing. By leveraging a fully digital generation method for LFM signals implemented on the ZCU111 FPGA platform, we carry out an validation test of the frequency-comparison monopulse, demonstrating its effectiveness in addressing the limitations of conventional monopulse.

1. Introduction

The adoption of wideband (WB) radar waveforms greatly enhances radar surveillance by improving range resolution, enabling high-resolution range profiling (HRRP), and supporting ISAR-based imaging. However, accurately estimating the Direction of Arrival (DoA) in WB systems remains a challenge. Traditional narrowband (NB) techniques such as amplitude- and phase-comparison monopulse rely on a Δ/Σ ratio analysis using two antennas, but suffer from angular ambiguity—where increased antenna spacing improves sensitivity but reduces unambiguous angular range—and from dependence on a single dominant scatterer, which limits performance for extended targets with multiple reflections. In contrast, this work introduces a frequency-comparison monopulse technique tailored for WB radar, leveraging the time-to-frequency mapping of Linear Frequency Modulated (LFM) signals to provide accurate and unambiguous DoA estimation, regardless of antenna spacing. A validation test was performed using the ZCU111 FPGA platform to digitally synthesize two LFM chirps emulating antenna-separated echoes. This controlled setup offers a cost-effective and flexible approach for demonstrating the technique's effectiveness, laying the groundwork for future experimental trials.

The remainder of the work is organized as follows. Section II introduces the frequency-based monopulse technique. Section III discusses the FPGA generation of LFM signals. Section IV presents the validation results. Finally, Section IV draws conclusions and outlines future work.

2. Frequency-comparison monopulse

The reference geometry for the frequency-based monopulse technique proposed in this paper is shown in Figure 1. Specifically, we consider two antennas at distance d, observing a target echo arriving from θ_0. Due to their spatial separation, the antennas receive the echo at slightly different time instants. We denote with $\Delta T = d \sin \theta_0 / c$ the delay between the antennas

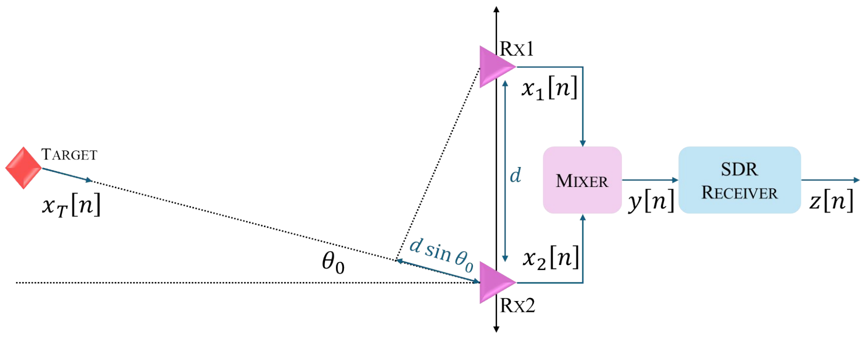

Figure 1 – Reference geometry of the frequency-comparison monopulse.

In discrete-time notation, the signals $x_1[n]$ and $x_2[n]$ at the two receiving antennas are:

$$x_1[n] = cos\left[2\pi f_{on}\left(1-\frac{B_r}{2}\right)n + \frac{\pi}{N} f_{on} B_r n^2 + \psi_0\right] \tag{1}$$

$$x_2[n] = x_1[n-n_0] = cos\left[2\pi f_{0n}\left(1-\frac{B_r}{2}\right)(n-n_0) + \frac{\pi}{N} B_r f_{0n}\ (n-n_0)^2 + \psi_0\right], \tag{2}$$

where:

- $f_{on} = f_0/f_s$ is the carrier frequency f_0 normalized to the sampling frequency f_s;
- $B_r = B/f_0$ is the fractional bandwidth;
- $n = 0 \ldots N-1$ is the discrete time index and N is the pulse length in samples;
- $n_0 = \Delta T f_s$ is the delay in samples between the two antennas;
- ψ_0 is the initial phase of the chirp.

A key property of LFM signals is that time delays map directly into frequency shifts through dechirping. Thus, the DoA can be estimated by measuring the frequency shift between the two signals after dechirping. Therefore, by mixing $x_1[n]$ and $x_2[n]$ we obtain the product signal:

3. FPGA generation of delayed Wideband Chirp Waveforms

To validate the proposed technique, we digitally synthesize two coherent LFM signals, x1[n]x_1[n] and x2[n]x_2[n], on the ZCU111 FPGA, emulating antenna reception without physical hardware. Following Pedersen's approach, a digital counter and accumulator generate a quadratic phase, mapped to an LFM waveform via a LUT for precise and repeatable signal control. We use a Numerically Controlled Oscillator (NCO), which combines a phase accumulator and LUT, driven by linearly increasing phase increments to produce chirps. The required phase offsets and increments for x1[n]x_1[n] and x2[n]x_2[n] are derived from their respective phases, isolating the time-independent and time-varying components.

$$\begin{cases} \phi_1^{offset} = \psi_0 \\ \phi_2^{offset} = \psi_0 + \frac{\pi}{N} B_r f_{0n}\ n_0^2 - 2\pi f_{0n}\left(1-\frac{B_r}{2}\right) n_0 \end{cases} \tag{7}$$

To derive the phase increments, $\Delta\phi_1[n]$ and $\Delta\phi_2[n]$, we define $\phi_1^{(o)}[n]$ and $\phi_2^{(o)}[n]$, which only include the terms of $\phi_1[n]$ and $\phi_2[n]$ which depend on n:

$$
\begin{cases}
\phi_1^{(o)}[n] = \dfrac{\pi}{N} f_{on} B_r n^2 + 2\pi f_{on}\left(1 - \dfrac{B_r}{2}\right) n \\
\phi_2^{(o)}[n] = \dfrac{\pi}{N} B_r f_{0n} n^2 + 2\pi f_{0n}\left[1 - \dfrac{B_r}{2} - \dfrac{B_r}{N} n_0\right] n
\end{cases} \tag{8}
$$

Note that (8) can be written recursively as

$$
\begin{cases}
\phi_1^{(o)}[n+1] = \phi_1^{(o)}[n] + \Delta\phi_1[n] = \phi_1^{(o)}[n] + \dfrac{\pi}{N} f_{on} B_r (2n+1) + 2\pi f_{on}\left(1 - \dfrac{B_r}{2}\right) \\
\phi_2^{(o)}[n+1] = \phi_2^{(o)}[n] + \Delta\phi_2[n] = \phi_2^{(o)}[n] + \dfrac{\pi}{N} f_{on} B_r (2n+1) + 2\pi f_{on}\left(1 - \dfrac{B_r}{2} - \dfrac{B_r}{N} n_0\right)
\end{cases} \tag{9}
$$

Since NCOs operate with a fixed number of bits, phase increments and offsets must be scaled accordingly. For an NCO with NaccN_{\text{acc}} bits, the phase ϕ[n]\phi[n] is converted into a digital value z[n]z[n] via z[n]=2Nacc2πϕ[n]z[n] = \frac{2^{N_{\text{acc}}}}{2\pi} \phi[n]. The chirp rate μ\mu is derived from the sampling frequency fsf_s as μ=fs22M\mu = \frac{f_s^2}{2^M}, and assuming M=NaccM = N_{\text{acc}}, this simplifies implementation. The counter then linearly steps from 0 to N−1N-1, where NN is the number of chirp samples.

For implementation on the ZCU111 FPGA, we used the Simulink System on Chip Blockset for high-level design. The chirps x1[n]x_1[n] and x2[n]x_2[n] are generated at baseband and upconverted using the FPGA's internal mixer, enabling wide bandwidth generation at moderate clock rates. With a 500 MHz clock, we set fs=500f_s = 500 MHz, allowing chirp bandwidths up to 200 MHz. Upconversion to a 900 MHz carrier yields a 22% fractional bandwidth. Using Nacc=16N_{\text{acc}} = 16 bits, the generated chirp duration is τp=52 μs\tau_p = 52\ \mu\text{s}.

This setup synthesizes signals as if received by two antennas spaced by distance dd and observing a target at angle θ0\theta_0, enabling realistic and controllable testing of the DoA estimation method.

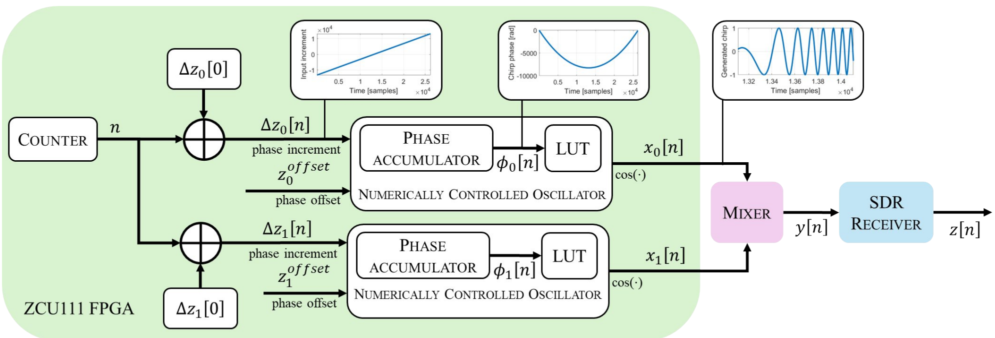

Figure 2 – Digital LFM generator, including the linear increment, the quadratic phase, and the generated chirp.

4. Processing Chain and Experimental Results

To validate DoA estimation, we avoided separate dechirping since no real radar scene was involved. Instead, two LFM chirps were directly mixed using a Mini-Circuits ZX05-43-s+ analog mixer (750–4200 MHz), merging dechirping and beat frequency extraction into one step. The resulting beat frequency was used to estimate the target's DoA, as defined in equation (6). The beat signal was captured at the mixer output using a USRP-2955 SDR.

Since the USRP-2955 operates in a frequency range of 10 MHz – 6 GHz the chirp received at the second antenna was generated with a higher carrier frequency: $f_0^{(0)} = 900MHz$ and $f_0^{(1)} = 1000MHz$. The chirp at $f_0^{(1)}$ is also shifted by $\Delta f = \mu d \sin\theta_0 / c$ because of the geometry.

After mixing, the resulting signals produce:

1. A chirp with bandwidth $2B = 400\ MHz$, centred at $f_0^{(0)} + f_0^{(1)} + \Delta f = 1900\ MHz + \Delta f$
2. A beat frequency tone at $f_0^{(1)} + \Delta f - f_0^{(0)} = 100MHz + \Delta f$.

The residual carrier at $f_{IF} = 100MHz$ is then downconverted to baseband by the internal I/Q demodulator of the USRP-2955, enabling the beat frequency tone at Δf to be measured from the data. Figure 3(a), (b) show respectively the time-domain representation of the beat signal at the mixer output and its power spectrum for antenna spacing $d = 3m$ and DoA of $\theta_0 = 60°$.

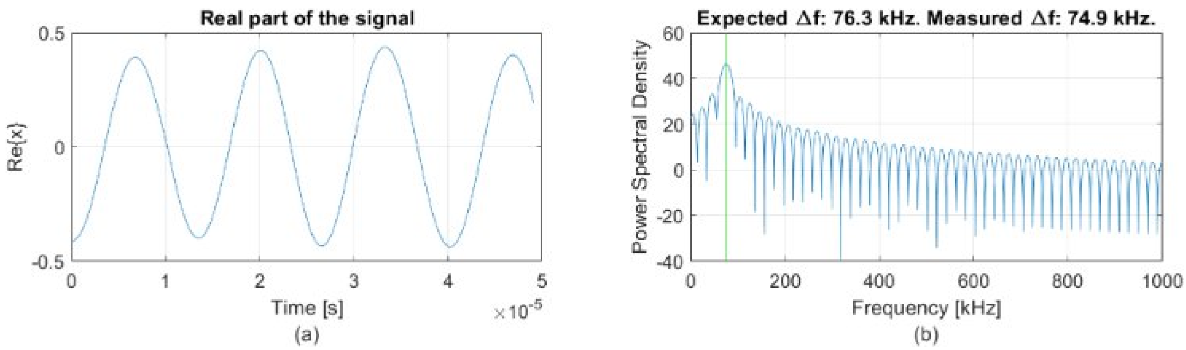

Figure 3 – (a) Time-domain representation of the beat signal. (b) Power spectral density of the beat signal.

The measured beat frequency is Δf^=74.9\hat{\Delta f} = 74.9 kHz, yielding an estimated angle θ^0=58.8∘\hat{\theta}_0 = 58.8^\circ, closely matching the true angle θ0=60∘\theta_0 = 60^\circ. The 1.5 kHz frequency mismatch is due to a carrier frequency offset (CFO) between the ZCU111 and USRP-2955, introducing a slight bias in angle estimation. This CFO can be characterized and pre-compensated to enhance accuracy.

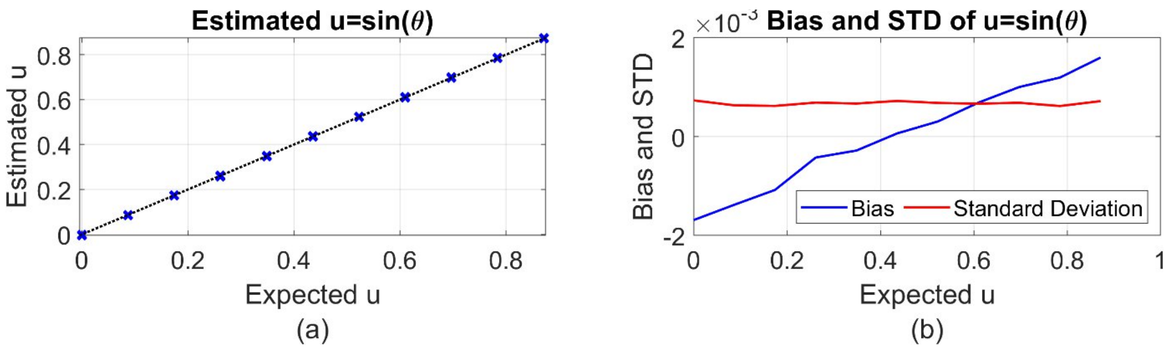

Figure 4 – (a) Expected and estimated values of $u_0 = sin\,\theta_0$, (b) Bias and standard deviation of $u_0 = sin\,\theta_0$.

5. Conclusions

This paper presents the first experimental validation of a frequency-based DoA estimation method for wideband radar using LFM signals. Implemented on the ZCU111 FPGA in a controlled lab setup, the technique demonstrated high accuracy and robustness. Results confirm that frequency-based monopulse offers reliable angle estimation, overcoming angular ambiguities and noise sensitivity common in traditional methods. Future work will focus on extending validation to open-field scenarios, addressing real-world far-field conditions.